JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 18, No. 5, September–October 1995

Proportional Navigation and Weaving Targets

Paul Zarchan*
Charles Stark Draper Laboratory, Inc., Cambridge, Massachusetts 02139

Sinusoidal or weave maneuvers on the part of the target can make it particularly difficult for a pursuing missile to hit. Normalized design curves are presented so that the guidance system designer can easily assess the influence of weave maneuvers on missile system performance. These curves show how the target weave frequency and amplitude along with the interceptor guidance system time constant and effective navigation ratio determine the size of the miss distance. Acceleration saturation effects also are considered as a further important influence on system performance. Finally, suggestions are made showing how to improve system performance.

Introduction

IT is well known that large miss distances can be induced by the target if a maximum acceleration maneuver is initiated at the proper time to go before intercept.[1–3] Against a missile utilizing a proportional navigation guidance system the best time for the target to maneuver depends on the missile guidance system time constant and effective navigation ratio. If the target does not have a priori knowledge of the missile parameters, does not know time to go until intercept, or does not have visual or electronic contact with the pursuing missile then periodic maneuver sequences offer the target the best chances of survival.[4–6] The barrel roll or weave maneuver is one such periodic maneuver sequence. It is also well known that tactical ballistic missiles (TBMs) can spiral or weave into resonance (i.e., TBM roll rate equals vehicles natural pitch frequency) as they re-enter the atmosphere due to either mass or configurational asymmetries.[7,8] Both the intentional weave maneuver on the part of an aircraft or the unintentional spiral maneuver on the part of a TBM make them both challenging targets from an interceptor point of view.

The paper will first study the influence of the target weave maneuver on a simplified single time constant proportional navigation guidance system. Closed-form solutions for the miss distance as a function of the effective navigation ratio, guidance system time constant, and weave maneuver amplitude and frequency will be derived. Since it is well known that the single time constant guidance system seriously underestimates the miss distance, a more realistic, higher order guidance system will be used to develop normalized miss distance design curves using the normalization factors from the single time constant target maneuver miss distance solutions. Next, it will be shown that the insights gained from the single time constant solution still apply to the higher order system. The finite acceleration capability of the interceptor also plays an important role in determining system performance. The previous normalized design curves, which assumed infinite missile acceleration capability, are modified to show how the missile acceleration advantage over the target plays a key role in determining system performance. Finally, methods for improving missile system performance against weaving targets will be discussed.

Step Maneuver in Single Time Constant Guidance System

The simplest possible guidance system in which there will be miss distance due to a maneuvering target is shown in Fig. 1. In this single time constant guidance system, missile acceleration n_L is subtracted from target acceleration to form a relative acceleration. After two integrations we have relative position y, which at the end of the flight is the miss distance $y(t_F)$. A division by range (or the closing velocity multiplied by the time to go until intercept) yields the geometric line-of-sight angle λ where the time to go is defined as

$$t_{go} = t_F - t \tag{1}$$

The missile seeker, which is represented in Fig. 1 as a perfect differentiator, effectively provides a measurement of the rotation rate of the geometric line-of-sight from the interceptor to the target. A noise filter whose dynamics are neglected in Fig. 1 processes the noisy seeker measurements to provide better estimates of the line-of-sight rate. A guidance command n_c is generated, based on the proportional navigation guidance law, from this noise filter output. The flight control system attempts to maneuver the missile to follow the desired acceleration command. The dynamics of the flight control system are modeled as a single lag with time constant T. Since the only dynamics in the guidance system of Fig. 1 are that of the flight control system, this block diagram is also known as a single time constant guidance system.

Using the method of adjoints, it can be shown that the miss (in units of feet) due to a step target maneuver n_T (in units of gravity) in a single time constant guidance system can be expressed in the Laplace transform domain as[1,3]

$$\frac{\text{miss}}{n_T}(s) = \frac{32.2}{s^3}\left[\frac{s}{s+1/T}\right]^{N'} \tag{2}$$

where T is the guidance system time constant and N' is the effective navigation ratio in the proportional navigation guidance law. Taking the inverse Laplace transform of the preceding expression yields miss distance solutions in the time domain where time is interpreted as homing time t_F or the time to go before intercept at which the target maneuvers:

$$\left.\frac{\text{miss}}{n_T}\right|_{N'=3} = 16.1 t_F^2 e^{-t_F/T} \tag{3}$$

$$\left.\frac{\text{miss}}{n_T}\right|_{N'=4} = 32.2\, t_F^2\left(0.5 - \frac{t_F}{6T}\right)e^{-t_F/T} \tag{4}$$

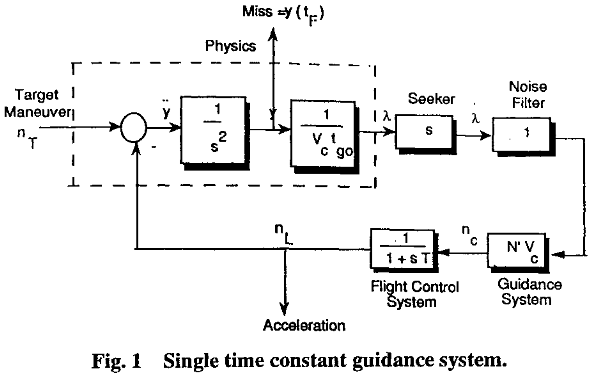

Fig. 1 Single time constant guidance system.

Received Aug. 25, 1994; revision received Dec. 19, 1994; accepted for publication Dec. 21, 1994.

*Principal Member of Technical Staff. Associate Fellow AIAA.

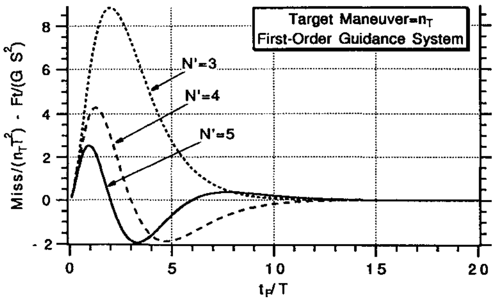

Fig. 2 There is no miss due to a step maneuver if the number of guidance time constant is greater than 10.

$$\left.\frac{\text{miss}}{n_T}\right|_{N'=5} = 32.2\,t_F^2\left(0.5 - \frac{t_F}{3T} + \frac{t_F^2}{24T^2}\right)e^{-t_F/T} \tag{5}$$

We can see from the preceding expressions that the exponential dependence of the miss on flight time and the reciprocal of guidance system time constant will cause the miss to approach zero when either the flight time gets very large or the guidance system time constant gets very small. The preceding closed-form normalized miss distance solutions due to a step target maneuver are displayed in Fig. 2 as a function of effective navigation ratio N'. We can see that if the ratio of the flight time to system time constant (also known as the number of guidance time constants) is greater than 10, the miss approaches zero. Otherwise the miss can be quite large. For example, when the guidance system time constant is 1 s and the effective navigation ratio is 3 we can see that the maximum miss distance of 9 ft due to a 1-g step target maneuver is induced when the flight time is approximately 2.5 s. We can also interpret the abscissa of Fig. 2 as the time to go before intercept at which the target maneuvers, normalized by the missile guidance system time constant. For the same numerical example this means that a 1-g maneuver at 2.5 s to go also yields a miss distance of 9 ft for any homing time greater than 2.5 s.

Weave Maneuver in Single Time Constant Guidance System

Periodic maneuver sequences such as the weaving target present a challenge for a proportional navigation missile guidance system. One possible planar representation of a weaving target is given by

$$\text{target maneuver} = n_T \sin \omega_T t \tag{6}$$

where n_T is the maneuver amplitude, ω_T the target weave frequency, and t time. The miss due to a weaving target as a function of flight time can be found either using the adjoint simulation method[1,3] or the method of brute force. For example, the miss distance results for a weaving 1-g target maneuver with weave frequency 1 rad/s appears in Fig. 3 where the missile guidance system time constant is 1 s ($T = 1$ s). Note that the abscissa of the graph is time of flight. We can see from Fig. 3 that unlike the step target maneuver case, the miss distance due to weaving target does not approach zero as the homing time increases. Depending on the flight time, the miss distance for this example can be as large as 11 ft or as small as zero when the effective navigation ratio is 3. Also note that after an initial transient period the miss is sinusoidal in nature with frequency 1 rad/s (i.e., the same as the target) regardless of effective navigation ratio. In addition, the maximum miss due to a weave maneuver at this particular weave frequency is slightly larger than that due to a step maneuver (i.e., 11 ft compared to 9 ft when $N' = 3$, 8 ft compared to 4 ft when $N' = 4$ and 5 ft compared to 2 ft when $N' = 5$).

Closed-form solutions for the miss due to a weaving target can be obtained in the steady state (i.e., at large flight times when transient dies out) by using simple electrical engineering techniques. It is easy

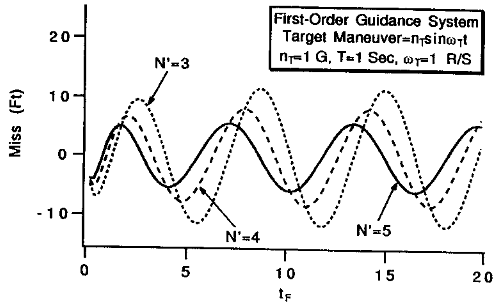

Fig. 3 Weaving target can induce a miss distance, even when the homing time is large.

to show that the miss due to a weave maneuver can be expressed in terms of Laplace transform notation as[1,3]

$$\frac{\text{miss}}{\text{weave}\,n_T}(s) = \frac{32.2}{s^2}\left[\frac{s}{s + 1/T}\right]^{N'} \tag{7}$$

where n_T is the maneuver magnitude, g. If a linear system has a sine wave input with frequency ω_T rad/s, the output in the steady state will also be a sinusoid of the same frequency but of different magnitude and phase. From basic steady-state electrical engineering circuit analysis techniques it can be shown that the magnitude and phase of the sinusoid output can be found by replacing s with $j\omega_T$ in the preceding transfer function and then finding the magnitude and phase of the resultant complex transfer function.[9] For example, if the effective navigation ratio is 3, the complex weave miss distance transfer function can be derived from the preceding equation by substitution as

$$\left.\frac{\text{miss}}{\text{weave}\,n_T}\right|_{N'=3}(j\omega_T) = \frac{32.2 j\omega_T}{(j\omega_T + 1/T)^3} \tag{8}$$

The magnitude and phase of this complex transfer function can be written by inspection as

$$\text{magnitude}|_{N'=3} = \frac{32.2\omega_T}{\left(\omega_T^2 + 1/T^2\right)^{1.5}} \tag{9}$$

$$\text{phase}|_{N'=3} = (\pi/2) - 3\tan^{-1}\omega_T T \tag{10}$$

Therefore the steady-state miss distance due to a weaving target can be written in the time domain as

$$\left.\frac{\text{miss}}{\text{weave}\,n_T}\right|_{\substack{N'=3\\ \text{steady state}}} = \text{magnitude}|_{N'=3}\sin(\omega_T t_F + \text{phase}|_{N'=3}) \tag{11}$$

or

$$\left.\frac{\text{miss}}{\text{weave}\,n_T}\right|_{\substack{N'=3\\ \text{steady state}}} = \frac{32.2\omega_T}{\left(\omega_T^2 + 1/T^2\right)^{1.5}} \times \sin\left(\omega_T t_F + \pi/2 - 3\tan^{-1}\omega_T T\right) \tag{12}$$

Figure 4 presents two cases in which the influence of a weave target maneuver on guidance system performance is evaluated. The first case assumes the target weave frequency is 1 rad/s whereas the missile guidance system time constant is 1 s, and the second case considers the target weave frequency to be 2 rad/s and the missile guidance system time constant to be 0.5 s. The miss distance results as a function of flight time generated with computer simulation (i.e., brute force or adjoint simulation techniques) are represented by dashed curves whereas the miss distance closed-form solutions obtained from the preceding formulas are represented with solid lines. We can see that after an initial transient period the closed-form steady-state miss distance solutions and computer results are in excellent agreement.

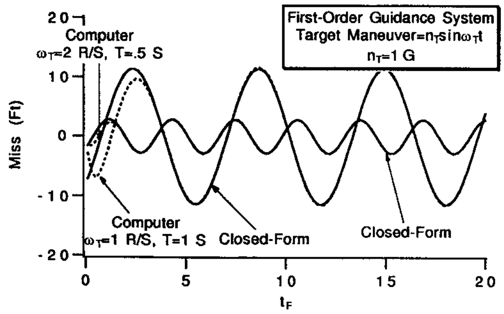

Fig. 4 Closed-form miss distance solutions agree with simulation results.

We have shown mathematically and by simulation that the miss distance due to a weaving target is a sinusoidal function of the flight time. Therefore, it is really a matter of luck on how large or small the miss distance will be. Of particular concern to the missile guidance system designer is the maximum or peak value of the sinusoidal miss distance function. The peak value of the miss due to a weave maneuver is simply the magnitude of the steady-state miss distance sinusoid. Therefore, the peak miss due to a weave maneuver in a single time constant proportional navigation guidance system with an effective navigation ratio of 3 is given by

$$\left.\frac{\text{peak miss}}{\text{weave}\, n_T}\right|_{N'=3} = \frac{32.2\omega_T}{\left(\omega_T^2 + 1/T^2\right)^{1.5}} = \frac{32.2\omega_T T^3}{\left(1+\omega_T^2 T^2\right)^{1.5}} \tag{13}$$

Using the same miss distance normalization factor as was used in the step target maneuver case the preceding expression becomes

$$\left.\frac{\text{peak miss}}{\text{weave}\, n_T T^2}\right|_{N'=3} = \frac{32.2\omega_T T}{\left(1+\omega_T^2 T^2\right)^{1.5}} \tag{14}$$

Letting x be the normalized target maneuver frequency where

$$x = \omega_T T \tag{15}$$

the peak miss distance formula simplifies further to

$$\left.\frac{\text{peak miss}}{\text{weave}\, n_T T^2}\right|_{N'=3} = \frac{32.2x}{(1+x^2)^{1.5}} \tag{16}$$

Similar expressions can be found for the peak miss distance due to a weave maneuver when the effective navigation ratios are 4 and 5 and can be shown to be

$$\left.\frac{\text{peak miss}}{\text{weave}\, n_T T^2}\right|_{N'=4} = \frac{32.2x^2}{(1+x^2)^{2}} \tag{17}$$

$$\left.\frac{\text{peak miss}}{\text{weave}\, n_T T^2}\right|_{N'=5} = \frac{32.2x^3}{(1+x^2)^{2.5}} \tag{18}$$

Figure 5 graphically displays the preceding formulas and shows how the steady-state normalized peak miss distance varies with the normalized target maneuver frequency (i.e., product of the target weave frequency and the missile guidance system time constant). We can see from Fig. 5 that the peak miss distance is close to a maximum when the normalized target maneuver frequency is near unity. Large weave frequencies do not cause much miss distances because very little target displacement is created. Small weave frequencies look like step target maneuvers and, thus, in the steady state (large flight times) cause very little miss distance. If we were on a collision triangle with the target (i.e., no heading error) and we coasted to the target by turning off the guidance (i.e., $N' = 0$), the peak miss distance would simply be the peak displacement n_T/ω_T^2 caused by the weaving target. Superimposed on Fig. 5 is the peak displacement or induced miss distance with no missile guidance (i.e., $N' = 0$) caused by the weaving target. We can see that for the single time constant guidance system, guiding with proportional navigation always yields a smaller miss against a weaving target than

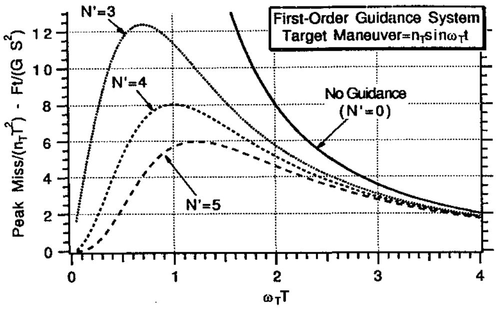

Fig. 5 Peak miss distance is maximum when normalized weave frequency is near unity.

coasting without guidance. However, for large values of normalized weave frequency the miss distance with and without guidance is approximately the same!

To illustrate the use of the normalized miss distance curves of Fig. 5, let us consider a numerical example in which there is a 6-g weaving target with a weave frequency of 2 rad/s. Assuming that the missile guidance system time constant is 1 s and effective navigation ratio is 3 we first compute the normalized weave frequency as

$$\omega_T T = 2 * 1 = 2 \tag{19}$$

which results in a normalized miss of approximately 5.5. Therefore, from the ordinate of Fig. 5 we can compute the peak steady-state miss distance to be

$$\text{peak miss} \approx 5.5 n_T T^2 = 5.5 * 6 * 1^2 = 33\,\text{ft} \tag{20}$$

Reducing the guidance system time constant to 0.5 s changes both the normalized weave frequency and the normalized miss. The new normalized weave frequency is

$$\omega_T T = 2 * 0.5 = 1 \tag{21}$$

which results in an increased normalized miss of approximately 11.5. However, the new peak steady-state miss distance is reduced because the guidance system time constant has been halved or

$$\text{peak miss} \approx 11.5 n_T T^2 = 11.5 * 6 * 0.5^2 \approx 17\,\text{ft} \tag{22}$$

Keeping the guidance system constant fixed to 0.5 s but increasing the target weave frequency to 4 rad/s increases the normalized weave frequency back to 2 or

$$\omega_T T = 4 * 0.5 = 2 \tag{23}$$

which again results in a normalized miss of approximately 5.5. The new peak steady-state miss distance becomes

$$\text{peak miss} \approx 5.5 n_T T^2 = 5.5 * 6 * 0.5^2 \approx 8\,\text{ft} \tag{24}$$

Thus, we can see that both the guidance system time constant and target weave frequency are important factors in determining the peak steady-state miss distance.

Higher Order Guidance System Dynamics

The single time constant guidance system model, used in the previous section, was useful because it could be used to derive closed-form solutions for the miss distance due to a weave maneuver. The single time constant guidance system miss distance formulas also suggest normalization factors for the miss distance. However, the disadvantage of the single time constant representation of a missile guidance system is that the miss distance can be seriously underestimated.

An endoatmospheric interceptor guidance system consists of a seeker, noise filter, and flight control system. Usually a minimum of five time constants (one for the seeker, one for the noise filter, and three for the flight control system) are required to realistically express the guidance system transfer function. If accurate information on guidance system dynamics is lacking, it is often useful to choose a canonic guidance system form so that preliminary design

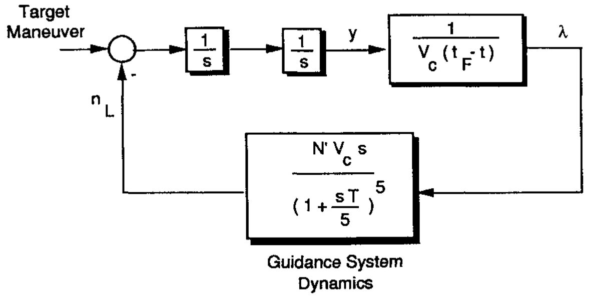

Fig. 6 Fifth-order binomial guidance system.

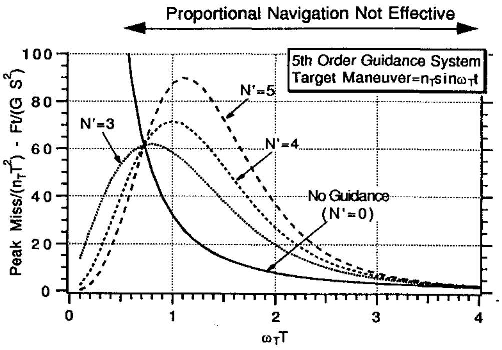

Fig. 7 Steady-state peak miss due to weave maneuver is much larger with fifth-order binomial guidance system.

and evaluation can take place. The binomial representation (i.e., all equal time constants) of the guidance system is the simplest possible since only one parameter, the guidance system time constant, provides all of the necessary information. In the limit, as the order of the binomial transfer function approaches infinity, the guidance system will act as a pure delay. Typically, the miss distances resulting from the binomial guidance system assumption will be conservative in the sense that it may yield slightly larger miss distances then will other guidance system transfer functions of the same order. The canonic fifth-order binomial guidance system transfer function is given by[1,10]

$$\frac{n_L}{\lambda} = \frac{N'V_c s}{(1 + sT/5)^5} \tag{25}$$

where T is the total guidance system time constant, n_L is the achieved missile acceleration, and λ is the line-of-sight angle. It is easy to show that with this canonic guidance system model, the overall guidance system time constant is simply the sum of the five individual time constants or T. The fifth-order binomial missile homing loop is shown in block diagram form in Fig. 6.

Figure 7 shows how the steady-state normalized peak miss distance due to a weave maneuver varies with the normalized target weave frequency for the fifth-order binomial guidance system. The curves in this figure are similar in shape to the ones of Fig. 5 but, as expected, the normalized miss distances are much larger. It is interesting to note the steady-state peak miss distance is still maximum when the normalized target weave frequency is approximately unity. Superimposed on Fig. 7 is the zero guidance miss distance or peak displacement n_T/ω_T^2 caused by the weaving target. Surprisingly, we can see that for the fifth-order guidance system, proportional navigation only yields a smaller miss than turning off the guidance system (i.e., $N' = 0$) when the normalized weave frequency is less than 0.7 (i.e., $\omega_T T < 0.7$). In other words, for normalized weave frequencies greater than 0.7, the weaving target nullifies the effectiveness of a proportional navigation guidance system!

To illustrate the use of the normalized miss distance curves of Fig. 7, let us reconsider the numerical example of the previous section in which there is a 6-g weaving target with a weave frequency of 2 rad/s. Assuming that the missile guidance system time constant is 1 s and effective navigation ratio is 3 we first compute the normalized weave frequency as from Eq. (19), which results in a normalized miss of approximately 20. Therefore, we can compute the peak steady-state miss distance to be

$$\text{peak miss} \approx 20 n_T T^2 = 20 * 6 * 1^2 = 120\,\text{ft} \tag{26}$$

which is four times larger than the miss in a single time constant guidance system (i.e., 120 ft vs 33 ft). Reducing the guidance system time constant to 0.5 s changes both the normalized weave frequency and the normalized miss. The new normalized weave frequency is computed from Eq. (21) which results in an increased normalized miss of approximately 60. The new peak steady-state miss distance becomes

$$\text{peak miss} \approx 60 n_T T^2 = 60 * 6 * 0.5^2 \approx 90\,\text{ft} \tag{27}$$

which is five times larger than the miss in a single time constant guidance system (i.e., 90 ft vs 17 ft). Keeping the guidance system time constant fixed to 0.5 s but increasing the weave frequency to 4 rad/s increases the normalized weave frequency back to 2 or after computation by Eq. (23) again results in a normalized miss of approximately 20. The new peak steady-state miss distance becomes

$$\text{peak miss} \approx 20 n_T T^2 = 20 * 6 * 0.5^2 \approx 30\,\text{ft} \tag{28}$$

which is approximately 4 times larger than the miss induced with a single time constant guidance system (i.e., 30 ft vs 8 ft). Thus, we can see that the higher order guidance system dynamics of the fifth-order binomial guidance system yield much larger miss distances due to a weaving target than does the single time constant representation of the guidance system.

Acceleration Saturation

We have observed in the preceding two sections that both the guidance system dynamics and effective navigation ratio play an important role in determining the miss distance due to a weaving target. The finite acceleration capability of the interceptor is also important in determining the miss distance. Normalized miss distance curves can also be developed when missile acceleration saturation effects are considered. In this case it is hypothesized that miss distance normalization factors remain unchanged but new curves have to be developed for the nondimensional ratio of the missile to target acceleration advantage or

$$\text{ratio} = n_{\text{lim}}/n_T \tag{29}$$

where n_{lim} is the interceptor acceleration limit.

Using the preceding ratio and the normalization factors for the steady-state peak miss due to a weaving target we can derive normalized miss distance curves by the method of brute force. In other words, we can generate normalized miss distance curves by simulating all of the possibilities. We can then infer performance by making extrapolations from the normalized miss distance curves. Of course, detailed checks have to be made to ensure that the normalization factors are correct. Figures 8–10 present the normalized steady-state peak miss distances due to a weaving target for effective navigation ratios ranging from 3 to 5, respectively. As expected, we can see that less missile acceleration capability (smaller ratio) means larger miss distances. In addition, we can also see by comparing Figs. 8–10 that increasing the effective navigation ratio tends to increase the miss

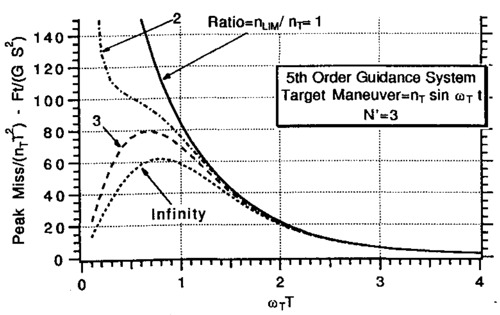

Fig. 8 Normalized steady-state peak miss due to weaving target and saturation effects for an effective navigation ratio of 3.

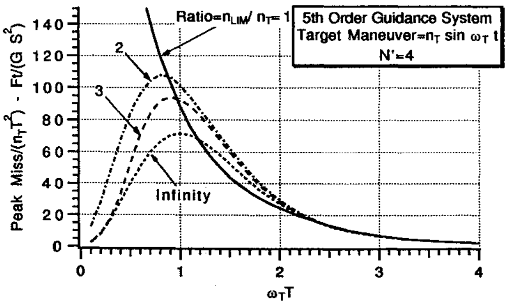

Fig. 9 Normalized steady-state peak miss due to weaving target and saturation effects for an effective navigation ratio of 4.

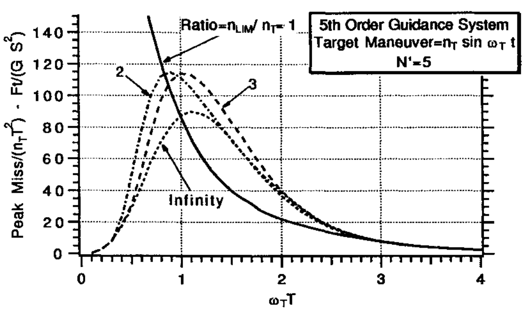

Fig. 10 Normalized steady-state peak miss due to weaving target and saturation effects for an effective navigation ratio of 5.

due to a weaving target. We can also see that the larger effective navigation ratios (i.e., $N' = 5$), increasing the missile acceleration capability may not always reduce the miss (i.e., $\omega_T T = 2$). Under these circumstances the weaving target is causing proportional navigation to be ineffective. This should not be surprising since we know that when the normalized weave frequency is greater than 0.7, doing nothing or $n_{\text{lim}}/n_T = 0$ is optimal.

To demonstrate the use of the normalized curves of Figs. 8–10 let us again consider the same example of the previous section in which there was a 6-g target weave maneuver with weave frequency of 2 rad/s and a proportional navigation missile guidance system with overall time constant of 0.5 s and effective navigation ratio of 3 (i.e., $n_T = 6$, $\omega_T = 2$, $T = 0.5$, and $N' = 3$). In this case the normalized weave frequency is 1 ($\omega_T T = 2*0.5 = 1$). If the missile acceleration limit is infinite then the ratio is infinite and we can read from Fig. 8 that the steady-state peak miss is 90 ft or

$$\text{peak miss}_{\infty G} = 60 n_T T^2 = 60 * 6 * 0.5^2 = 90\,\text{ft} \tag{30}$$

Reducing the acceleration limit to 18 g reduces the ratio to 3 or

$$\text{ratio} = n_{\text{lim}}/n_T = 18/6 = 3 \tag{31}$$

For a normalized weave frequency of 1 the new steady-state peak miss increases to 105 ft or

$$\text{peak miss}_{18G} = 70 n_T T^2 = 70 * 6 * 0.5^2 = 105\,\text{ft} \tag{32}$$

Reducing the acceleration limit further to 12 g reduces the ratio to 2 or

$$\text{ratio} = n_{\text{lim}}/n_T = 12/6 = 2 \tag{33}$$

For a normalized weave frequency of 1 the new steady-state peak miss increases to 113 ft or

$$\text{peak miss}_{12G} = 75 n_T T^2 = 75 * 6 * 0.5^2 = 113\,\text{ft} \tag{34}$$

Finally, reducing the acceleration limit even further to 6 g reduces the ratio to 1 or

$$\text{ratio} = n_{\text{lim}}/n_T = 6/6 = 1 \tag{35}$$

For a normalized weave frequency of 1 the new steady-state peak miss increases to 128 ft or

$$\text{peak miss}_{6G} = 85 n_T T^2 = 85 * 6 * 0.5^2 = 128\,\text{ft} \tag{36}$$

In this example, if the missile had no acceleration capability or if the guidance system was turned off, the peak miss would be the maximum value of the weave displacement n_T/ω_T^2 or only 48.4 ft.

If the target weave frequency was increased to 4 rad/s and everything else remained the same, the new normalized weave frequency would be doubled to 2 ($\omega_T T = 4 * 0.5 = 2$). In this case we can see from Fig. 8 that the miss is independent of the missile to target acceleration advantage and that the miss would reduce to 30 ft or

$$\text{peak miss}_{\infty G, 18G, 12G, 6G} = 20 n_T T^2 = 20 * 6 * 0.5^2 = 30\,\text{ft} \tag{37}$$

Again, turning the guidance system off would make the peak miss equivalent to the maximum value of the weave displacement n_T/ω_T^2 or only 12.1 ft.

Improving Performance

In general, the safest and most effective method for improving the performance of a proportional navigation guidance system against the weaving target is to reduce the overall guidance system time constant and to increase the missile to target acceleration advantage. In aerodynamically controlled missiles, the major contributor to the guidance system time constant is usually the flight control system time constant, and the limitation on missile acceleration capability is a function of the maximum angle of attack at which a missile can operate without causing flight catastrophe (i.e., instability due to pitch-yaw-roll cross coupling). The ability to speed up the missile flight control system and the challenge in increasing the missile's maneuverability depend on advances in flight control system technology. Aberration effects (known as radome effects for rf missiles) will set a lower limit on how small the missile flight control system time constant can be made without causing stability problems,[11] and flight control system pitch-yaw-roll cross coupling will place an upper limit on maximum permissible angle of attack. Although a thorough discussion of the challenges in speeding up a flight control system and safely achieving high angles of attack is beyond the scope of this paper, two numerical examples will be presented in this section showing the benefits to system performance if these goals can be met.

To illustrate the importance of reducing the guidance system time constant, a nonnormalized, nonsteady-state example was chosen in which there was a 6-g weaving target with a weave frequency of 2 rad/s. Figure 11 shows that the miss distance induced by a weaving target on a fifth-order binomial proportional navigation guidance system dramatically decreases with decreasing guidance system time constant. In fact, when the guidance system time constant is 0.1 s there is virtually no miss due to the weaving target.

If we fix the guidance system time constant at 0.1 s, we can see from Fig. 12 that although increasing the target weave frequency increases the miss, the miss is still small. We could also have calculated the maximum peak steady-state miss in this example from the normalized curves of Fig. 7. For an effective navigation ratio of 3, the curve of Fig. 7 is a maximum when the normalized weave frequency is 0.7. That means for this example the actual target weave

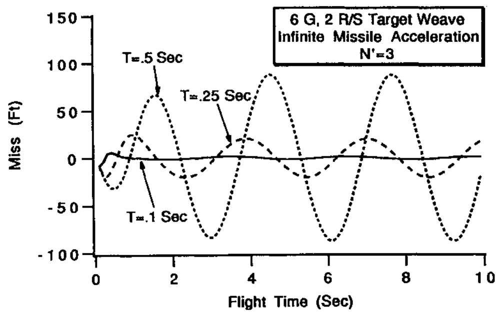

Fig. 11 Reducing guidance system time constant dramatically reduces miss.

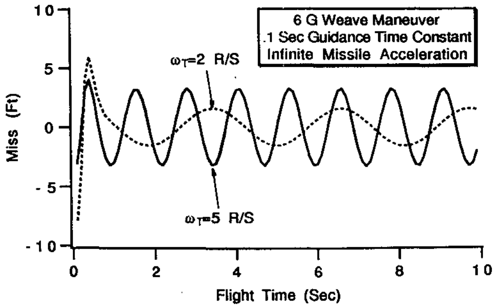

Fig. 12 Small guidance time constant yields good performance even when weave frequency increases.

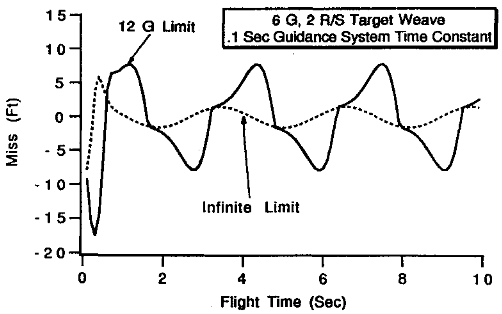

Fig. 13 Small miss distances can be achieved even when there is only 2 to 1 acceleration advantage.

frequency is 7 rad/s (i.e., $\omega_T T = 7 * 0.1 = 0.7$). From Fig. 7 we can see that the actual maximum peak miss is approximately 4 ft or

$$\text{peak miss}|_{N'=3} = 63 n_T T^2 = 63 * 6 * 0.1 \approx 4\,\text{ft} \qquad (38)$$

Thus, we can see that a very small miss distance can be achieved against this difficult maneuver if the guidance system time constant can be reduced to 0.1 s. Of course, we can also see from Fig. 7 that turning the guidance system off would also yield the same miss.

Both previous examples assumed that the missile had infinite acceleration capability. Figure 13 shows that when the missile to target acceleration advantage decreases from infinity to only 2 the miss increases. However, since the guidance system time constant is small the maximum miss distance is not large. Of course, if the missile guidance system were turned off the miss would only be approximately 4 ft regardless of acceleration limit.

Conclusions

Normalized design curves have been presented showing how a weaving target influences the miss distance of a generic proportional navigation guidance system. The paper demonstrates that the target weave frequency and amplitude, the missile guidance system time constant, effective navigation ratio, and acceleration capability all play an important role in determining system performance. Targets with very low weave frequency appear as near-constant target maneuvers and cause no problem for a proportional navigation guidance system provided that the number of guidance time constants remaining after the initiation of the maneuver is at least 10. Targets with very high weave frequencies also cause no problem for a missile guidance system because there is very little resultant target displacement as a result of the maneuver. In between these two target weave frequency extremes the miss distance will increase. The paper shows that when the product of the target weave frequency and missile guidance system time constant is approximately unity the resultant miss distance will be a maximum. Examples are presented showing that when the product of the target weave frequency and missile guidance system time constant exceeds 0.7 proportional navigation is not acting in an effective manner. It is demonstrated that, in general, speeding up a missile guidance system and increasing the missile to target acceleration advantage will help reduce the miss distance due to a weaving target.

Acknowledgments

This paper is based on work that was sponsored by Massachusetts Institute of Technology Lincoln Laboratories under Prime Contract F19628-C-0002. In particular I would like to thank Richard Mozzicato, Stephen Weiner, and Keh-Ping Dunn for their advice and valuable suggestions.

References

[1]Zarchan, P., *Tactical and Strategic Missile Guidance*, Progress in Astronautics and Aeronautics, Vol. 124, AIAA, Washington, DC, 1990, Chap. 5, pp. 96–98.

[2]Shinar, J., and Steinberg, D., "Analysis of Optimal Evasive Maneuvers Based on a Linearized Two-Dimensional Model," *Journal of Aircraft*, Vol. 14, No. 8, 1977, pp. 795–802.

[3]Travers, P., "Interceptor Dynamics," unpublished lecture notes, Raytheon, Bedford, MA, circa 1971.

[4]Zarchan, P., "Representation of Realistic Evasive Maneuvers by the Use of Shaping Filters," *Journal of Guidance and Control*, Vol. 2, No. 4, 1979, pp. 290–295.

[5]Forte, I., and Shinar, J., "Can a Mixed Guidance Strategy Improve Missile Performance," *Journal of Guidance, Control, and Dynamics*, Vol. 11, No. 1, 1988, pp. 53–59.

[6]Imado, F., and Miwa, S., "Missile Guidance Algorithm Against High-g Barrel Roll Maneuvers," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 1, 1994, pp. 123–128.

[7]Platus, D. H., "Ballistic Reentry Vehicle Flight Dynamics," *Journal of Guidance, Control, and Dynamics*, Vol. 5, No. 1, 1982, pp. 4–16.

[8]Chadwick, W. R., "Reentry Flight Dynamics of the Non-Separating Tactical Ballistic Missile," Proceedings of AIAA/BMDO 3rd Annual Interceptor Technology Conf., San Diego, CA, July 1994.

[9]Javid, M., and Brenner, E., *Analysis, Transmission and Filtering of Signals*, McGraw–Hill, New York, 1963, Chap. 6, pp. 140–166.

[10]Nesline., F. W., and Zarchan, P., "Miss Distance Dynamics in Homing Missiles," *Proceedings of AIAA Guidance and Control Conference*, AIAA, New York, 1984, pp. 84–98.

[11]Nesline., F. W., and Zarchan, P., "Radome Induced Miss Distance in Aerodynamically Controlled Homing Missiles," *Proceedings of AIAA Guidance and Control Conference*, AIAA, New York, 1984, pp. 99–115.